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WEAK COUPLING LIMIT AND GENUINE QCD PREDICTIONS FOR HEAVY QUARKONIA*

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ABSTRACT

Although individual levels of toponium will be unobservable, the top–anti–top system near threshold fulfills all requirements of a rigorous perturbation theory in QCD for weakly bound systems. Corresponding techniques from positronium may thus be transferred successfully to this case. After clarifying the effect of a non-zero width we calculate the $t\bar{t}$ potential to be used for the calculation of e.g. the cross-sections near threshold.

Perturbative quantum field theory by means of the general Bethe–Salpeter (BS) formalism is applicable also for weakly bound systems, described as corrections to a zero order equation for the bound system. The latter is usually taken as the Schrödinger–equation, but experience in positronium has shown that other starting points of perturbation theory, like the Barbieri-Remiddi (BR) equation,[1] offer advantages. That such rigorous methods [2] have not found serious considerations for a long time in quarkonia has to do with the importance of confinement effects in QCD. Parametrizing the latter by a gluon-condensate [3], one finds that for quark masses m up to about 50 GeV nonperturbative effects are of the same order as corrections to the bound states [4], athough the situation seems a little less serious using BS-methods[5]. Nevertheless, the possible practical advantages of a rigorous approach have been demonstrated a long time ago, not for level shifts in bound-states, but for perturbative corrections to the decay of quark-antiquark systems. Straightforward applications of perturbative QCD to the annihilation part *alone* of, say, bottom-anti-bottom, with minimal subtraction at a scale $O(2m)$ gave huge a correction $O(10\alpha_s/\pi)^6$. However, in such a decay the bound quark pair is really off-shell. Although from that it is clear that an (off-shell) annihilation part by itself cannot even be gauge independent: The

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perturbative correction to the wave-function of the decaying state must be added for consistency. In fact, after doing this and performing a careful renormalization procedure at Bohr momentum $O(\alpha_s m)$, appropriate for that system, it could be shown that large corrections tend to compensate in that result, at least in the case of the singlet ground state (0^{-+}) [7].

Now, with a top quark in the range of 150 - 180 GeV [8] for the first time confinement effects are negligible. The large width Γ of the decay $t \to b + W$ makes bound–state 'poles' at real energies unobservable, but at the same time obliterates confinement effects even *above* threshold because the top quark has no time to 'hadronize'. The effect of Γ can be taken into account by simple analytic continuation of the zero order equation to complex energies [9]. Previous work [10] was based upon phenomenological quarkonium potentials. However also BS perturbation theory can be applied to the $t\bar{t}$ Green function G near the poles which move into the unphysical sheet of the complex plane at a distance Γ to the real energy axis. Nevertheless, the residues and hence the wave functions together with the QCD level shifts remain real. Hence they can be used in a straightforward manner to determine the different contributions to a rigorous QCD potential.

After showing that this analytic continuation argument also works for the BR-equation, we determine the different contributions for such a quantity from the (real) energy shifts, including (numerical) $\mathcal{O}(m\alpha_s^4)$–effects. With a generic perturbation H to the Green function G_0 of the zero order equation, the level shifts become [2]

$$\Delta E_n = \langle\langle h_i \rangle\rangle(1 + \langle\langle h_1 \rangle\rangle) + \langle\langle h_0 g_1 h_2 \rangle\rangle + \mathcal{O}(h^3) \tag{1}$$

where h_i and g_i are the expansion coefficients of H, resp. G_0 near the pole $E \sim E_n$ of G_0. The expectation values are *four*–dimensional momentum integrals, taken here with respect to BR wavefunctions. The latter differ by factors produced by relativistic corrections and with p_0 from (normalized) Schrödinger–wavefunctions. Nevertheless, we formulate our final result as a 'potential'. The full formula for the different parts of

$$V = \sum_{i=0}^{2} V_{QCD}^{(i)} + V_{EW} \tag{2}$$

can be found in ref.[11]. (i) refers to the loop order which, of course, is not uniquely correlated with orders of α_s in energy shifts. — $V_{QCD}^{(0)}$ beside the Coulomb exchange contains the $\vec{p}^{\,4}$–term and the exchange of a transversal gluon, producing the 'abelian' relativistic corrections $\mathcal{O}(m\alpha_s^4)$. We *do not* include a running coupling constant anywhere because this would mix orders, spoiling even the gauge–independence order by order within any application. Of course, for technical reasons eq.(2) is derived and should be applied in the Coulomb gauge.

$$V_{QCD}^{(1)} = -\frac{33\alpha^2}{8\pi r}(\gamma + \ln \mu r) + \frac{\alpha^2}{4\pi r}\sum_{j=1}^{5}[\mathrm{Ei}(-r m_j e^{\frac{5}{6}}) - \frac{5}{6} + \frac{1}{2}\ln(\frac{\mu^2}{m_j^2} + e^{\frac{5}{3}})] + \frac{9\alpha^2}{8mr^2}$$

consists of vacuum polarization and vertex corrections. The first contain the gluon loop and the loops from fermions. Although the level contributions will be $\mathcal{O}(m\alpha_s^3)$

for the toponium system the mass of the charm and bottom must not be neglected, because they yield *numerical* $\mathcal{O}(m\alpha_s^4)$ corrections. In contrast to the QED case here also the one–loop gluon–splitting vertex yields a potential $\propto \alpha_s^2/mr^2$, important to this order. In the vacuum polarization part of

$$\begin{aligned} V_{QCD}^{(2)} &= c^{(H)}\frac{4\pi\alpha^3}{r} \\ &\quad -2\frac{\alpha^3}{(16\pi)^2 r}\left\{(33-2n_f)^2[\frac{\pi^2}{6}+2(\gamma+\ln\mu r)^2]+9(102-\frac{38}{3}n_f)(\gamma+\ln\mu r)\right\} \end{aligned}$$

we emphasize the importance of *non-leading* logarithms which would only be contained in the usual running coupling constant if a two–loop β–function would be used. Among the 'box' corrections, an H–graph (with the figure H formed by gluons between the two fermion lines) is emphasized as an contribution which gives at least a correction to the Coulomb term. In

$$\begin{aligned} V_{EW} &= -\frac{8}{9}\alpha_{QED}(\mu)\frac{4\pi\alpha}{r}-\sqrt{2}G_F m^2\frac{e^{-m_H r}}{4\pi r}+\sqrt{2}G_F m_Z^2 a_f^2\frac{\delta(\vec{r})}{m_Z^2}(7-\frac{11}{3}\vec{S}^{\,2}) \\ &\quad +\sqrt{2}G_F m_Z^2 a_f^2\frac{e^{-m_Z r}}{2\pi r}[1-\frac{v_f^2}{2a_f^2}-(\vec{S}^{\,2}-3\frac{(\vec{S}\vec{r})^2}{r^2})(\frac{1}{m_Z r}+\frac{1}{m_Z^2 r^2})-(\vec{S}^{\,2}-\frac{(\vec{S}\vec{r})^2}{r^2})], \end{aligned}$$

photon–exchange, Z–exchange and Z–anihilation turn out to be as essential for the $t\bar{t}$–system as the usual relativistic corrections [11].

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